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THE SPACE BETWEEN HELMHOLTZ AND EINSTEIN: MORITZ SCHLICK ON SPATIAL INTUITION AND THE FOUNDATIONS OF GEOMETRY[0]

1. BIOGRAPHICAL INTRODUCTION

Comparable to Rudolf Carnap who – in his last letter to Otto Neurath in 1945[1] – named Neurath the "big locomotive" of the "Vienna circle-train", Moritz Schlick can perhaps be called both the *designing engineer* and the *engine-driver* of this train: From 1924 to 1929, the Vienna group around Schlick was the germ of what *later* became known as the Vienna school of logical empiricism or neo-positivism and shaped its philosophical programme considerably. Carnap, in his autobiography, remarked that Schlick's *Allgemeine Erkenntnislehre* (Schlick, 1918) anticipated a number of the circle's later philosophical achievements and formed the nucleus of many of its formal elaborations. On the other hand, Schlick's assassination in 1936 marked the end of the public period, i.e., the organised activities and broad perception of the Vienna circle. Beyond doubt, *he* was the one who kept successfully integrated the divergent philosophical and ideological tendencies of the group, and therefore became the "father figure" of the movement.

This paper will concentrate on *one* period and *one* aspect of Schlick's work that can perhaps best illustrate the "Interaction between Mathematics, Physics and Philosophy" within *early* logical empiricism *before* Vienna: The development of Schlick's understanding of *intuition*, especially of *spatial intuition*, in connection with the scientific development of his *early* career. The main object is not a systematic critique of his concept of intuition, but a historical investigation of its changing meaning and relevance in Schlick's scientific thinking. As the structure of this paper is (by and large) shaped by his different publications at different times from 1910 to 1921, it will start with a few *biographical* and *bibliographical* remarks:

Schlick was educated as a physicist in Berlin. As one of his doctor-father M. Planck's favourite disciples[2] he wrote his thesis *Über die Reflexion des Lichtes in einer inhomogenen Schicht* (Schlick, 1904), which became his first publication. Schlick then turned to philosophy. As a rare exception within the Vienna-circle he did not confine his interests to philosophy of science and epistemology, but also published on ethics and practical philosophy in general (see, for instance, his "Lebensweisheitslehre" (Schlick, 1908) and "Fragen der Ethik" (Schlick, 1930b)). His first publications concerning our subject, however, include his *Habilitationsschrift*, published as "Das Wesen

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der Wahrheit nach der modernen Logik" (Schlick, 1910) and his essay "Gibt es intuitive Erkenntnis?" (Schlick, 1913).

The *second* part of this paper presents a discussion of these and other early publications of Schlick. The focus will be on Schlick's understanding of *intuition* as a guarantee of evidence and certainty of our knowledge (a subject which is, of course, of special importance to mathematics). This critical survey will end with Schlick's main philosophical work (Schlick, 1918 [Engl. 1974]) *Allgemeine Erkenntnislehre* ("General Theory of Knowledge").

Next to Philipp Frank, Schlick was the first member of the Vienna Circle who recognized the philosophical relevance of Einstein's two theories of relativity and tried to work out its implications with respect to space and the foundations of geometry. Starting with "The Philosophical Significance of the Principle of Relativity" (Schlick, 1915), he published from 1915 onwards a number of papers on this subject which Einstein himself at several times praised as the best *philosophical* interpretations of his ideas at all.[3] Well known became Schlick's booklet "Space and Time in Contemporary Physics" (Schlick, 1917), which achieved four editions from 1917 to 1922. In this year (1922) he succeeded Mach and Boltzmann on the chair for the Philosophy of the Inductive Sciences at the University of Vienna. Even more than Frank's papers, "Space and Time in Contemporary Physics" made relativity theory popular in early logical empiricism. Without doubt, Schlick very much contributed to appoint Einstein (next to Russell and Wittgenstein) to one of the three "leading representatives of the scientific world view", as they were called in the Vienna circle-programme from 1929.[4] Schlick's interpretation of Einstein's theories – as far as his concept of space and his understanding of geometry is concerned – will be discussed in the *third* part.

Though Schlick must have started to study Helmholtz sometime before 1915 (see (Schlick, 1915, p. 150)), his first and only detailed discussion of Helmholtz's views on geometry is from 1921, the one-hundredth anniversary of his birth. On this occasion, Schlick and Paul Hertz published Helmholtz's *Schriften zur Erkenntnistheorie* (Helmholtz, 1921). Schlick himself added extensive and detailed comments on two of Helmholtz's most important papers concerning our subject: "Über den Ursprung und die Bedeutung der geometrischen Axiome" (Helmholtz, 1870) and "Die Tatsachen in der Wahrnehmung" (Helmholtz, 1878). In 1922, a collection of lectures on Helmholtz, given on the same occasion, appeared; they include Schlick's "Helmholtz als Erkenntnistheoretiker" (Schlick, 1922). His comments and critical discussions will be shortly examined in the *fourth* part, though *intuition* in his *early* period will remain the main subject.

2. SCHLICK'S BASIC EPISTEMOLOGICAL PROBLEM: INTUITION, EVIDENCE AND CERTAINTY OF KNOWLEDGE

From the very beginning, Schlick's discussion of *intuition in general* is linked to what he later calls the "fundamental problem of epistemology" or the "problem of absolutely certain knowledge"[5]: In the history of philosophy, intuition is usually regarded not only as a source of knowledge, but as a source of evident, indubitable, and *certain* knowledge. And this, of course, will be the link to Schlick's discussion of geometry: The claim both of traditional rationalism as well as of Kantian apriorism is that *this* kind of privileged knowledge is realized in *mathematics*, and that its essential features – evidence and certainty – can be transferred to *empirical* knowledge by applying mathematics.

Already Schlick's paper "On the Nature of Truth in Modern Logic" (Schlick, 1910), contains the central argument that he will later use repeatedly in order to demolish what he labels the "theory of evidence"[6], i.e., the view that true knowledge consists entirely in or depends essentially on an immediate and *evident* experience called intuition (*Anschauung*). Schlick does not deny evidence as a psychological entity, and he agrees with Wilhelm Wundt's attitude that "all immediate evidence has intuition as its source" (Schlick, 1910, p. 441). But the claims of the "theory of evidence" in question go further: According to it, evidence is the decisive (if not the only) criterion of truth. Against *this* view Schlick argues that evidence is nothing but a subjective feeling underlying complex and opaque psychological influences not accessible to objective control. As the history of philosophy and the history of science have shown, supposed evidence has frequently led to erroneous (and even absurd) conclusions.

In order to save evidence in the light of such experiences as a meaningful concept, it would have to be disentangled from certainty (*Gewißheit*): Now, one might say that a proposition that seemed to be evident, but later turned out to be wrong was only experienced as (or felt) to be *certain* (and not as truly *evident*). According to Schlick, two possibilities arise from this expected loophole: Either true evidence and deceiving certainty are experienced as basically different. This would mean that a criterion exists that distinguishes evidence and certainty—a criterion, of course, which may not be *post hoc*, in order to avoid circularity of the argument.

But in this case deception by evidence (*Evidenztäuschung*) could not happen at all, and the introduction of an attribute "certainty" *different* from the attribute "evidence" would be entirely meaningless.

Or, on the other hand, the experience of evidence and the experience of certainty are *not* basically different. In this case only *later* investigations may reveal whether certainty *with* evidence or certainty *without* evidence was experienced. But this possibility would admit that the immediate experience of evidence is no criterion of truth *at all*, and that such a criterion has to be found in *later* experiences. But *these* can not be experiences of evidence in order to avoid circularity. This means that the separation of evidence and certainty fails in this case, too. To sum up Schlick's argument: Evidence can be *no* criterion of truth, and there is no immediate experience or intuition as a source of truth available to us, either.[7]

Evidence is also unnecessary as an *additional* characteristic of some propositions next to truth. Schlick asks: "What point is there in establishing self-evidence if we can verify the truth of a judgement directly by the presence of its essential features"? (Schlick, 1918, p. 131; [Engl. 1974], p. 150). This is a problem of his theory of verification, where he strongly parallels factual truths (*Tatsachenwahrheiten*) and conceptual truths (*Begriffswahrheiten*) (Schlick, 1910, pp. 435–458, esp. p. 445). Of special importance in this context is his criterion of truth for logical and mathematical propositions or, as Schlick puts it, for conceptual truths. Here we find that evidence still plays a more prominent role than he is willing to admit in his former criticism of the so-called 'theory of evidence'.

If we desist from the question of truth-transfer to propositions by deduction and restrict our attention to the truth of first premises or *axioms*, Schlick's view seems to be roughly this:

Axioms can not be characterized by a conceptual necessity (*begriffliche* [...] *Notwendigkeit*) (Schlick, 1910, p. 441). (Euclid's parallel postulate, for example, is not necessary in this sense, as is shown by the existence of non-Euclidean geometries.) Axioms can neither be characterized by evidence in the sense of an immediate experience of their truth. They *need to be* exemplified, but they also *can* be exemplified, and this is the point where *intuition* becomes important. The exemplification consists in the translation of general and abstract concepts into concrete and intuitive ideas (*Vorstellungen*). Only at *this* level intuition can gain evidence and clarity (Schlick, 1910). The truth of an axiom, however, is not comprehended by an immediate experience of evidence of this kind, but by a more complicated process of *identification*:

The general proposition is applied to an intuitive example, which gives occasion for an immediate experience of a certain fact (*unmittelbar erlebter Tatbestand*) which is expressed in a second proposition. And it is the identity of both propositions that brings about evidence and verifies the axiom. Schlick indeed draws a parallel between verification of factual propositions by outward experience and verification of logical and mathematical propositions by intuition, which he also labels "inward experience", "immediate experience" or "perfect experience" (Schlick, 1910, pp. 447f.). But this parallel stresses their *process character* rather than their *outcome*: Verification of axioms does not mean a *foundation* of their truth in inner experience (comparable to verification of a factual proposition), but rather a *recognition* (*Konstatierung*) of their truth in inner experience. Schlick *sharply separates* the epistemological status of the results in both cases:

> [...] full certainty [...] is not possible in the case of factual propositions, [...] because their verification depends on perception, in short on the outward world, [whereby] its laws are never known to us completely and with certainty. In the case of propositions about the relations of concepts, however, no perceptions are needed; they are verified by immanent processes, so to speak by tools which always accompany our mind; they are known to it more perfect than the relations of real things, and that is why they can be absolutely true to [our mind]. (Schlick, 1910, p. 447)

Though Schlick appeals to the works of Hilbert, Russell and Couturat and indicates that Kant's theory of intuition and mathematical certainty will no longer be acceptable in the light of their logical foundation of mathematics, his own understanding of so-called 'conceptual propositions' shows some Kantian reminiscences . In agreement with this seems to be, for example, his support of Poincaré's view that mathematical *induction* will never be reducible to logic, but is rather a necessity of our thinking, rooted in the nature of our understanding: Here, too, "we find the ‚eternal verities' founded in inner or perfect experience" (Schlick, 1910, p. 453).[8]

But over the next few years, Schlick's assessment of intuition and his philosophy of mathematics in general will change, and the more or less implicit 'Kantian connection' of intuition and the truth of conceptual propositions will vanish. Within this process two different phases can be separated: The first one (from about 1913 to 1917), is a more 'destructive', the second one (from 1918 onwards) is a more 'constructive' period. Schlick's reception of Einstein's doctrine seems to me of interest for both periods, though his departure from intuition obviously has internal epistemological reasons, too.[9] Relativity theory, he remarks in his paper "Die philosophische Bedeutung des Relativitätsprinzips" with respect to Kant's understanding of spatial and temporal intuition, "forces us to wake up from a little dogmatic slumber" (Schlick, 1915, p. 153).

The more 'destructive' period: Already in his paper "Gibt es intuitive Erkenntnis?" from 1913 Schlick makes clear "that the deepest insights of the present time, especially in theoretical physics have shown that now and again the intuitive representation [of knowledge] has to be abandoned, just in order to preserve knowledge in its whole purity" (Schlick, 1913, p. 485). He has not only spatial intuition in mind here, which becomes his main subject with Einstein's general theory of relativity from 1915 onwards. His general point is that Einstein's special theory of relativity violates certain presumingly intuitive and evident assumptions of Newtonian absolute space and time, and he now sharply criticises Kant for trying to give a philosophical foundation for these assumptions by mixing up conceptual knowledge of physical objects with the pure intuition of space and time. (Schlick, 1913, p. 485) It seems to be a plausible conjecture that *via* his reception of Einstein's special theory of relativity *Kant* became the main target of his reflexions on intuition from 1913 onwards, while other philosophers mentioned (as Husserl and Bergson, for example) were of secondary importance.

Schlick's destruction of intuition as an epistemologically relevant concept is linked to his earlier definition of knowledge and his sharp *demarcation* of knowledge and intuition: Knowledge is always conceptual; I *know* an object if I designate it by concepts in a univocal manner, and then integrate these concepts into the whole net of concepts and judgements already established. Hence, knowledge *always* demands two elements: something that is recognized (*etwas, das erkannt wird*) and the thing as *what* it is recognized (*dasjenige, als was es erkannt wird*) (Schlick, 1913, p. 485). Knowledge is neither an immediate experience nor a mental representation of reality, but consists entirely in the unambigious relation between objects and concepts.[10] These concepts are no images or pictures but mere *signs* – an understanding of concepts which is strongly influenced by Helmholtz's theory of signs.[11] Though Schlick's reception and modification of this theory is beyond the scope of this paper, some implications concerning his understanding of *intuition* should be considered:

First, it would contradict the essentially *symbolic* character of knowledge[12] to assume something like *intuitive knowledge*. Intuition "survives" only as a (more or less immediate) experience (*Erleben*) of a single object, and has nothing to do with the comparing, ordering and relating activity that gains proper knowledge. Therefore, Schlick's (short and concise) conclusion runs as follows: "*Intuitive* knowledge is *a contradictio in adiecto*." (Schlick, 1913, p. 481)

Schlick does *not* dissociate himself explicitly from his *former* use of intuition, which was meant to secure the truth of the propositions of logic and mathematics. He just mentions casually that the talk of "intuitive knowledge" might be perhaps justified in those propositions where (in the process of thinking) concepts can be represented by intuitive ideas (*anschauliche*

Vorstellungen). (Schlick, 1913, p. 484f.) But there is no allusion to (or revival of) his earlier view that truth and evidence of logical or mathematical axioms might be gained by the pretty weird process of identification sketched earlier. Quite contrary, Schlick insists on the purely *psychological* character of this kind of intuition, which is "comparable to the colouring in order to display the details of microscopical objects" (Schlick, 1913, p. 485). And he emphasizes that modern developments, especially in physics, have shown that fundamental new insights sometimes require the *renunciation* of intuitive support.

I call this first period *destructive* because intuition is deprived of any epistemological relevance. It is, as he says, "quite the opposite of knowledge" (Schlick, 1913, p. 486). But on the other hand, Schlick obviously sticks to the conviction that the propositions of pure mathematics and the mathematical sciences must be somehow epistemologically privileged, i.e., characterized by truth and (at least in the first area) by certainty. With intuition lost, they can no longer be regarded as synthetic a priori in Kant's sense. But what else can be their distinctive characteristic? With respect to this problem, Schlick's papers from 1913 onwards are rather disappointing, and it is not before his publication of *Allgemeine Erkenntnislehre* in 1918 that he treats it in a more *constructive* manner. But even during the period of destruction, Schlick's rejection of Kant's theory of intuition remains *ambiguous*:

In his discussion of Richard Hönigswald's interpretation of Einstein's theory of special relativity[13], given in his *Zum Streit über die Grundlagen der Mathematik* (Hönigswald, 1921) for example, Schlick comes to the conclusion that Kant's doctrine of space and time as forms of pure intuition has *not necessarily* to be abolished, but should be somehow *modified*:

He wants to deprive Kant's forms of intuition "from all quantitative, all mathematical, all metrical attributes"; as defining elements of a subjective, necessary and a priori form of intuition remain "only qualitative attributes of space and time"—or "in short the genuinely *temporal of time*, the specifically *spatial of space*" (Schlick, 1915, p. 163). But Schlick's discussion leaves *totally unclear* what these elements might be. Moreover, in the context of his whole epistemological framework purely *qualitative* forms of space and time can be nothing but psychological constructs without 'foundational output', and this is obviously *more* than a mere modification of Kant. All in all, Schlick's analysis of intuition during this period seems half-hearted and inconclusive.

The more 'constructive' period: Schlick's attempt in his *Allgemeine Erkenntnislehre* to save an area of *exact* and *certain* knowledge *without* recourse to intuition depends predominantly on his interpretation of what he (and others) describe as *'Hilbert's* method of implicit definition', i.e., the definition of the basic concepts of geometry by their axioms.[14] This approach is characterized by Schlick as a "path that is of the greatest significance for epistemology" (Schlick, 1918, p. 31; [Engl. 1974], p. 33).[15] His argument, which differs from Hilbert's original intention, can be summed up as follows: Knowledge needs objective and exact concepts instead of subjective and vague ideas. Explicit definitions of a concept rely on its attributes and in the end – in order to avoid a *regressus at infinitum* – on concrete or ostensive definitions of attributes. Therefore, they inevitably end in subjective and inexact immediate experiences (*Erleben*) (Schlick, 1918, p. 28; [Engl. 1974], p. 29). Schlick turns to the method of implicit definition in order to save (as he says) "the absolute certainty" and "rigor" of knowledge without any endangering appeal to intuition (Schlick, 1918, p. 29; [Engl. 1974], p. 30). Thus axiomatics, allegedly developed along Hilbert's line, becomes Schlick's model of scientific conceptualization – not only for geometry, but also for other branches of mathematics (as number theory) and even for the empirical sciences (especially theoretical physics).

Schlick's leading idea obviously is to relate the dichotomy of implicit and explicit definition to his older dichotomy of knowledge and intuition (where intuition includes both empirical intuition, *Erlebnis*, and Kant's pure intuition). Implicit definitions, according to *his* understanding, avoid any kind of intuition: They determine concepts completely and precisely through their logical relations which are fixed by a system of axioms (Schlick, 1918, p. 30ff.; [Engl. 1974], p. 31ff.). Schlick uses a nice picture in order to contrast explicit and implicit definitions:

> In the case of ordinary definitions, the defining process terminates when the ultimate indefinable concepts are in some way exhibited *(Aufzeigen)* in intuition [. . .] This involves pointing to something real, something that has individual existence. [. . .] In short, it is through concrete definitions that we set up the connection between concepts and reality. Concrete definitions exhibit in intuitive or experienced reality that which henceforth is to be designated by a concept. On the other hand, implicit definitions have no association or connection with reality at all; specifically and in principle they reject such association; they remain in the domain of concepts. A system of truths created with the aid of implicit definitions does not

> at any point rest on the ground of reality. On the contrary, it floats freely, so to speak, and like the solar system bears within itself the guarantee of its own stability. None of the concepts that occur in the theory designate anything real; rather, they designate one another in such fashion that the meaning of one concept consists in a particular constellation of a number of the remaining concepts. (Schlick, 1918, p. 35; [Engl. 1974], p. 37)

Schlick's *Allgemeine Erkenntnislehre* does not say very much about the question how to come from the 'solar system' down to the 'ground' of reality, or, to be less metaphorical, how to connect precise concepts and relations of the axiomatic system with concrete empirical objects or intuitive mathematical models and examples. This problem will soon be discussed in the context of *spatial* intuition. But first it is important to note the implications of Schlick's new approach to what he called the "fundamental problem of epistemology", i.e., the certainty and evidence of our knowledge—or at least of some parts of it. Schlick sums up his position in these words:

> [...] it is to this point that our consideration returns again and again – the moment we carry over a conceptual relation to intuitive examples, we are not longer assured of complete rigor. When real objects are given us, how can we know with absolute certainty that they stand in just the relations to one another that are laid down in the postulates through which we are able to define the concepts?
>
> Kant believed that immediate self-evidence assures us that in geometry and natural science we can make apodictically certain judgements about intuitive and real objects. For him the only problem was to explain how such judgements come about, not to prove that they exist. But we who have come to doubt this belief find ourselves in an altogether different situation. All that we are justified in saying is that Kantian explanation might indeed be suited to rendering intelligible an *existing* apodictic knowledge of reality; but *that* it exists is not something that we may assert, at least not at this stage of our inquiry. Nor can we even see at this point how a proof of its existence might be obtained. (Schlick, 1918, p. 36; [Engl. 1974], p. 38)

Two aspects of Schlick's conclusion seem to me especially important: *First*, Schlick claims that Kant's theory of mathematical intuition and evidence in its 'historical form' is no longer tenable; therefore his explanation of the applicability of mathematics to 'real world problems' is *obsolete*. He does not claim, however, that any new attempt to establish synthetic principles a priori will necessarily fail. This marks a strong contrast to his later and neither proven nor provable claim to have shown that synthetic principles a priori are "a logical impossibility" (*eine logische Unmöglichkeit*). (Schlick, 1930a [repr. 1938], p. 25)

Secondly, and more general, Schlick's sharp demarcation of concept and intuition, of logico-mathematical thinking and empirical reality 'shuts the door' for mathematical certitude and evidence when mathematics is applied to the realm of empirical phenomena. Exact and certain knowledge is restricted to formal properties, defined by the axiomatic system. Empirical content is always infected by the subjectiveness and uncertainty of immediate experience and intuition. That is the point of view soon adopted by Einstein in his lecture "Geometrie und Erfahrung" (Einstein, 1921) and summed up in his famous *dictum*: "As far as the propositions of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality" (Einstein, 1921, pp. 385f.). Schlick's final solution of his so-called 'fundamental problem of epistemology' is – with respect to intuition – definitely a *negative* one. Paraphrasing Einstein, one might say: As far as we can gain absolute certainty of knowledge, it does not depend on intuition, and as far as we have intuition at our disposal, it can not found knowledge at all.[16]

3. INTUITIVE SPACES AND CONCEPTUAL SPACE: EINSTEIN'S GENERAL THEORY AND SCHLICK'S RECEPTION

In the second edition of his *Allgemeine Erkenntnislehre* (Schlick, 1918, [^{2}1925]), Schlick takes into account Einstein's "Geometrie und Erfahrung".[17] He sums up his affirmative discussion as follows: "Geometrical space is a conceptual tool (*Hilfsmittel*) for designating the ordering of the real. There is no such thing as pure intuition of space, and there are no *a priori* propositions about space" (Schlick, 1918, [^{2}1925], p. 326; [Engl. 1974], p. 255). This statement draws attention to a fundamental problem in Schlick's concept of geometry: If the basic concepts of geometry are defined by an abstract and uninterpreted system of axioms, it is by no means clear how they should somehow contribute to a designation of – in his terms – 'the ordering of the real'. Schlick's attempt to solve this problem depends strongly on a reinterpretation of intuition:

For Schlick, subjective and private experience is *real*, and the question is how the subjective and private experience of *spatiality* and *temporality* of these data can be linked to objective concepts of space and time. This happens, as he says in his *Allgemeine Erkenntnislehre*, "always [...] in accordance with the same method, which we may call the *method of coincidences*. It is of the greatest significance epistemologically" (Schlick, 1918, p. 234; [Engl.], p. 272).[18] His leading idea concerning this method can be described as follows: We achieve intuitive experience of spatiality through our different senses, the visual sense, the tactile sense, etc. There are as many intuitive spaces as there are different senses, though these spaces are totally different in *qualitative* respect.[19] We can, however, experience at the same time a *singularity* in two different intuitive spaces. For example, I can look at the tip of my pencil and, at the same time, I can touch it with my finger. The result is a *coincidence* of two different singularities with different qualities. These two singularities are now coordinated to the same point in objective space by abstracting from the qualitative properties of the different spaces.[20] Other coincidences will yield other points in objective space, and the system of points gained can be extended to a continuous manifold by our thinking.

Though, generally speaking, Schlick perceives Einstein's general theory of relativity rightly as a scientific theory that fits quite well to the epistemological framework of the *Allgemeine Erkenntnislehre* and not as a theory that shaped its epistemological content, it seems that his understanding of the method of coincidences owes to it at least one insight: Einstein's general theory worked as a *mediator* in order to come from this *topological* concept of objective space to a *metrical* concept. Schlick insists that "all measurements, from the most primitive to the most advanced, rest on the observation of spatio-temporal coincidences" (Schlick, 1918, p. 236; [Engl.], p. 275). At the end, all measurements of physical magnitudes, not only distances or time intervals, are based on these spatio-temporal coincidences and thereby on coincidences of singularities in different intuitive spaces. At first sight, this seems to be nothing but an allusion to Einstein's early and frequently repeated claim that his special theory of relativity is a theory about coincidences of events (i.e., coincidences of world lines).[21] This could, in a way, even be said for Newtonian physics. But in his "Space and Time in Contemporary Physics" (Schlick, 1917). Schlick makes clear *what* the peculiarity of Einstein's *general* theory is: *Both* classical physics *and* the special theory of relativity sticked to the idea of an "Euclidean structure" of space – the first one by assuming the existence of a rigid rod for measurements of length, the second one by assuming that all measurements in a system can be performed by a rod which is at rest with respect to this system. The metrical determination was expected to be entirely independent of other physical conditions (of gravitational fields, for example). This preference of Euclidean metric, however, was removed by Einstein's general theory.[22] The elimination of a certain and fixed background geometry matches Schlick's idea how objective space (or rather space-time) is constructed when we proceed from our different intuitive spaces: The experienced coincidences of this *intuitive level* are first brought into a *topological* order. The spatio-temporal manifold thus reached is nothing but the embodiment or essence (*Inbegriff*) of objective elements defined by the method of coincidences. (Schlick, 1917 [^{3}1920], p. 83) Measurement and metric are, in a way, secondary: Measurement always *depends* on coincidences in space and time, and the metrical properties built into the laws of physics always *serve* the attempt to yield a general representation of the coincidences in space-time.[23] According to Schlick, this is the procedure by which we can construct mathematical models for a system of axioms of mathematical physics.[24] Two remarks may be appropriate in order to make this point more explicit:

First. It must be said that Schlick is not very clear or explicit about the question how the abstract level of implicit definitions at the top and the model-building at the ground are related in detail. In so far, his theory of coincidences seems to be unsatisfactory.

Second. As the restricting conditions of Riemannian metric are neither accepted as evident or even necessary by Schlick, *and* as the transition from coincidences to measurement (and metric) always is in need of physical specifications and, indeed, of arbitrary physical assumptions, (Schlick, 1917 [^{3}1920], pp. 55f.) Schlick rejects Cassirer's thesis that the very concept of the Riemannian line element includes synthetic a priori-knowledge of space. (Schlick, 1921, p. 101)[25]

Without going into the details of Schlick's criticism of Cassirer from 1921, his main point here as well as in other papers on relativity from this period can be summed like this: 'Space' is a medal with *two* sides, a *conceptual* one *and* an *intuitive* one. The *method of implicit definition* allows to define space by precise concepts. The *method of coincidences* gains an understanding of its intuitive basis. Kant was right in stressing both the conceptual and the intuitive side of space. But without the method of implicit definitions on the one hand and the method of coincidence on the other hand, he could not but mixing up conceptual and intuitive elements of space (in his fiction of space as a form of *pure intuition*).[26] Einstein, however, opened a new way of bringing back spatial intuition *without* mixing it up with conceptual knowledge. His general theory of relativity has shown that the same method of coincidence by which we proceed from empirical or psychological intuition to objective space underlies the physical construction of space.

According to Schlick, this is probably the most important epistemological outcome of Einstein's general theory.[27]

To refer back to Schlick's rather opaque 'modification-claim' discussing Einstein's theory of special relativity in 1915: Kant's approach should not be given up, but should be essentially modified; space should be freed from all quantitative attributes ('the specifically spatial of space'). His coincidence-argument from 1917 onwards can be understood as an elaboration of this idea: The empirical or psychological intuition of space became his *substitute* for Kant's pure intuition of space, though this substitute had not to carry on the foundational burdens of Kant's original conception.

4. SPATIAL INTUITION AND THE PROCESS OF CONCEPTUALIZATION: SCHLICK'S CRITICISM OF HELMHOLTZ'S APPROACH

1921 was a very fertile year in Schlick's career: He did not only discuss Cassirer's[28] and Reichenbach's[29] analysis of Einstein's theories, but also edited (with Paul Hertz) Helmholtz's *Schriften zur Erkenntnistheorie* (Helmholtz, 1921). In this context, Schlick commented two of Helmholtz's most important papers in some detail: "On the Origin and Significance of the Axioms of Geometry" (Helmholtz, 1870) and "The Facts in Perception" (Helmholtz, 1878). Helmholtz's investigations of the concepts of space and the foundations of geometry are closely linked to his research in the field of physiology, especially on visual and tactual perception. In short, it is *our own* mobility in space, our ability to occupy different positions with respect to perceived objects, that makes spatial localization of objects possible and is, indeed, decisive for our *concept of space* itself.[30]

Starting from the existence and free mobility of rigid bodies as a precondition of congruence and measurement, his principal aim (in the first instance) was to establish the Euclidean structure of physical space as an intuitively necessary concept. *In so far* he defended a Kantian-like understanding of spatial intuition: Though intuition is brought in by visual and tactual perception, it serves as a guarantee of the Euclidean character of physical geometry.

It is pretty clear that Schlick must have been sympathetic with the empirical origin of Helmholtz's intuition of space, but it is also clear that he could not accept its 'Euclidean services'. In the two papers commented by Schlick, however, Helmholtz changes his position considerably: The axioms of Euclidean geometry are no longer considered as being necessary by their foundation in a transcendental form of intuition. We can imagine other spaces with other axioms and we can – what is most important for Helmholtz's empiricist approach – imagine lawful sense-experiences in those spaces without contradictions. Intuition of space therefore can *not* yield the necessity of *any* system of geometrical axioms, and in so far Euclid's geometry becomes *contingent*. (Helmholtz, 1870 [repr. 1921], p. 22.)

This new consequence was, of course, most welcome to Schlick. To his mind there remains, however, an important inconclusiveness in Helmholtz' argument: Though spatial intuition can not yield any necessity of geometrical axioms, Helmholtz sticks to Kant's idea that there is something like space *as a subjective and transcendental form of intuition*. This seems to be Schlick's basic problem with Helmholtz's foundation of geometry. Three remarks about Schlick's criticism may be sufficient here:

First. According to Helmholtz, the really interesting features of spatial intuition are those which can *not* be grasped by axiomatic systems like Euclid's. After all, that is why he states that "space can be transcendental without the axioms being it", as he puts it in his famous *dictum*.[31] In this context, Helmholtz describes several sense experiences which come close to those of the *Allgemeine Erkenntnislehre*, and can therefore be integrated into Schlick's psychological intuition analysed by the method of coincidences. Consequently, they pose no problem for Schlick and are commented affirmatively.

Second, and perhaps more important: Helmholtz's intuition of space is *richer* than Schlick's, because it operates with the free mobility of rigid bodies and therefore includes the idea of *constant curvature* of space. Helmholtz makes quite clear that he does *not* misunderstand perfect rigidity as a meaningful *empirical* concept; for him it is rather a concept *a priori*, introduced in order to make measurement possible.[32] *And* at the same time we have to presuppose that "the measuring instruments which we take to be fixed, actually (*wirklich*) are bodies of unchanging form. Or that they at least undergo no kinds of distortion other than those which we know [...]" (Helmholtz, 1870 [repr. 1921], p. 18; [Engl. 1977], p. 19)

According to Schlick, Helmholtz's argument results from an inadmissible *extension* of spatial intuition or, to put it otherwise, from a confusion of intuition and conceptual knowledge. This is his indignant comment on Helmholtz's last sentence:

> In the little word 'actually' there lurks the most essential philosophical problem of the whole lecture [of Helmholtz]. What kind of sense is there in saying of a body that it is *actually* rigid? According to Helmholtz's definition of a fixed body

> [...], this would presuppose that one could speak of the distance between points 'of space' without having regard to bodies; but it is beyond doubt that without such bodies one can not ascertain and measure the distance in any way. [...] If the content of the concept 'actually' is to be such that it can be empirically tested and ascertained, then there remains only the expedient [...] to declare those bodies to be 'rigid' which, when used as measuring rods, lead to the *simplest* physics. Those are precisely the bodies which satisfy the condition [of coincidences] adduced by Einstein. Thus what has to count as 'actually' rigid is then not determined by a logical necessity of thought or by intuition, but by a convention, a definition. (Helmholtz, 1870 [repr. 1921], p. 33, n. 40; [Engl. 1977], p. 34).[33]

In short, Helmholtz's idea of constructing space on the basis of the free mobility of rigid bodies is *totally* rejected by Schlick. Helmholtz's most important 'intuitive link', i.e., the relation of a priori-rigidity and of empirical measurement – was obviously 'cut off' for Schlick by *general relativity*. And, considered the other way round: Schlick applied the philosophical lesson he drew from Einstein's theory to Helmholtz's approach, i.e., sharply to separate abstract concepts and intuitive experience – thereby, however, he failed to appreciate Helmholtz's mathematical contribution to the foundations of geometry.

Third, and probably most important, there is a profound difference in the epistemological perspective of Helmholtz and Schlick that has to be considered: Schlick's residual of spatial intuition rests, as mentioned earlier, on immediate sense experience. Though it is linked to objective, physical space by his method of coincidence, the whole conceptual framework of objective space is built up *deductively* or 'top down', starting with axioms, because only the method of implicit definitions can guarantee *precise* basic concepts.

Helmholtz's approach is quite contrary. Gregor Schiemann describes it aptly as an *inductive* or 'bottom up'-conceptualization:[34] Axioms are of minor importance. We proceed from the perception of qualities, whereby the lawlikeness of spatial relations is brought in by our own free mobility. On *this* basis we build up spatial concepts by association or unconscious induction. Without going into the details of Helmholtz's argument, it can be said that according to his approach there can be no sharp separation of intuition and conceptual knowledge, as it is claimed by Schlick. Intuition can change by learning—we can learn, for example, how it would be to move in a space with negative curvature. Helmholtz points out the difference between his idea of intuition and the traditional one, which supposes a "flash-like evidence" of spatial intuition. (Helmholtz, 1878 [repr. 1921], p. 161) And against Kant he claims that "the most essential progress of science", especially physiology, was the decomposition of traditional intuition into "elementary processes of thinking" which, as he says later, "can not yet be expressed by words" (Helmholtz, 1878 [repr. 1921], p. 172). This is, of course, not compatible with Schlick's tidy separation of intuition and knowledge, and his comments on Helmholtz's writings on geometry reflect this 'dualism'.

5. CONCLUDING REMARK

The purpose of this paper was to light up Schlick's changing understanding of intuition as a result of an 'interaction between mathematics, physics and philosophy': Schlick's 'turning off' from a form of intuition, which is Kantian-like in so far as intuition guarantees the truth of the propositions of geometry, seems to be influenced by his reception of Einstein's theory of relativity, especially the method of coincidences. In addition, his *later* argument for a sharp separation of knowledge and intuition makes use of the method of implicit definition which he attributes to Hilbert's *Grundlagen der Geometrie*. Schlick's 'new' separation is directed both against Kant and Helmholtz. It may be asked, however, if he does justice to Helmholtz's approach, according to which spatial intuition can be dissolved into 'elementary processes of thinking'. This approach seems not to be so far away from Einstein's method of coincidences, by which Schlick wants to do justice to intuition: The 'Space between Helmholtz and Einstein' seems to be more restricted than he is willing to admit.

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NOTES

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In the following titles are abbreviated by names and the first date of appearance. If quotations are *not* from the first edition, the year of publication is added in square brackets. If an English translation is quoted, the bracket contains, in addition, the abbreviation "Engl.". In cases where the English short title is added in round brackets, the note (or reference) refers to the first (German) title or quotation.

[1]Letter from Carnap to Neurath from the 23.8.1945; the quotation is drawn from (Hegselmann, 1992, p. 23).

[2]See, for instance, (Heilbron, 1988, pp. 188f.).

[3]Cf. (Stadler 1997, pp. 201f.; Hentschel 1986, pp. 476ff.; Hentschel 1990, pp. 377f.; Howard 1984, pp. 618f.; Howard 1988, pp. 204ff.).

[4]Cf. (Carnap/Hahn/Neurath, 1929 [repr. 1999], pp. 166f.).

[5]Cf. (Schlick, 1934, pp. 79-80, 94-95).

[6]Cf. (Schlick, 1910, pp. 389ff.).

[7]Cf. (Schlick, 1910, pp. 390–392) – on p. 392 evidence is, strange enough, reintroduced not as a sufficient, but as a necessary feature of truth; (Schlick, 1918, pp. 130–131, 68–69).

[8]Cf. (Goldfarb, 1996, p. 214).

[9]Cf. (Schlick, 1918, p. 148f.) for his dissociation of his former theory of truth.

[10]Cf. (Friedman, 1999, p. 20).

[11]Cf. (Friedman, 1999, esp. p. 20).

[12]Cf. (Schlick, 1913, p. 481).

[13]Cf. (Howard, 1994, p. 51).

[14]It is well known that the notion 'implicit definition' was already introduced by J. D. Gergonne in 1818, and also, that this notion was, at the end of the 19th century, applied by the school of Peano for the *different* approach of definitions by postulates; see (Otero, 1969/70). F. Enriques and others soon applied it from 1904 onwards to Hilbert's axiomatics, because Hilbert explicitly stated that the axioms are *also definitions* of the basic concepts, without using the term 'implicit definition' (see (Hilbert, 1902, pp. 71–72; Gabriel, 1978) where the confusion of the older and newer meaning of the term by the Peano school and its impact on its later use is analysed). That Schlick's interpretation and adaptation of Hilbert's approach are problematic, was already pointed out by Majer (2001, pp. 214–216).

[15]Cf. (Goldfarb 1996, p. 214).

[16]Cf. (Schlick, 1918, p. 130; [Engl. 1974], p. 149).

[17]Cf. (Ferrari, 1994, p. 436, n. 64).

[18]Cf. (Ferrari, 1994, pp. 435f.).

[19]Cf. (Friedman, 1999, pp. 37f.).

[20]See (Schlick, 1918, p. 235f.; [Engl.], pp. 274f.); cf. (Schlick, 1917, p. 96f.).

[21]Cf. (Coffa, 1991, p. 198, 399, n. 6).

[22]Cf. (Schlick, 1917 [^{3}1920], pp. 46f.).

[23](Schlick, 1917 [^{3}1920], pp. 50f.) "Alle Weltbilder, die hinsichtlich der Gesetze jener Punktkoinzidenzen übereinstimmen, sind physikalisch absolut gleichwertig." (p. 51).

[24]Cf. (Friedman, 1999, p. 38).

[25]Cf. (Cassirer, 1921, p. 101; [repr.1957], p. 93).

[26]Cf. (Schlick, 1921, pp. 108–109).

[27]Cf. (Ryckman, 1992, pp. 494f.).

[28]Cf. (Ryckman, 1991; Ferrari, 1994).

[29]Cf. (Friedman 1994).

[30]Cf. (Helmholtz, 1868 [repr. 1921], pp. 38-40 (introductory part); 1866 [repr. 1883], pp. 610-612).

[31]Helmholtz's title of the "2. Beilage" from "Facts in Perception"; cf. (Helmholtz, 1878 [repr. 1921], p. 140).

[32]Helmholtz (1870 [repr. 1921], p. 18) discusses influences of temperature and other minor factors.

[33]Cf. (Helmholtz, 1870 [repr. 1921], p. 30, n. 31; [Engl. 1977], p. 31).

[34]See (Schiemann, 1997, p. 350) with respect to (Helmholtz, 1878 [repr. 1921], pp. 123f.).

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